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ALGEBRA

FACTOR / गुणनखंड

→ $(a+b)^2 = a^2+2ab+b^2$ → $(a-b)^2 = a^2-2ab+b^2$

→ $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$ → $(a+b)^2-(a-b)^2 = 4ab$

⇓ ⇓

$\frac{(a+b)^2+(a-b)^2}{(a^2+b^2)} = 2$ $\frac{(a+b)^2-(a-b)^2}{ab} = 4$

→ a^2+b^2+ab ⇒① $(a+b)^2-ab$ → a^2+b^2-ab ⇒① $(a+b)^2-3ab$

⇒② $(a-b)^2+3ab$ ⇒② $(a-b)^2+ab$

⇒③ $\frac{a^3-b^3}{(a-b)}$ ⇒③ $\frac{a^3+b^3}{(a+b)}$

→ $a^3+b^3 = (a+b)(a^2+b^2-ab)$ & $a^3-b^3 = (a-b)(a^2+b^2+ab)$

Q:→ $\frac{21^3-19^3}{21-19} + 21\times19 = ?$

Solution → let $a=21$ & $b=19$ then $\frac{a^3-b^3}{(a-b)} + a\times b$

⇒ $\frac{(a-b)(a^2+b^2+ab)}{(a-b)} + a\times b$ ⇒ $(a^2+b^2+ab)+ab$

⇒ $(a+b)^2$ ⇒ $(21+19)^2 = (40)^2 = 1600$ ans

⇒ from $a^3+b^3 = (a^2+b^2-ab)(a+b)$ से →

⇒ $\frac{a^3+b^3}{(a^2+b^2-ab)} = a+b$ व $\frac{a^2+b^2-ab}{a^3+b^3} = \frac{1}{a+b}$

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अब $a^3-b^3 = (a-b)(a^2+b^2+ab)$ से

$$\frac{a^3-b^3}{(a^2+b^2+ab)} = (a-b) \quad \text{व} \quad \frac{a^2+b^2+ab}{a^3-b^3} = \frac{1}{(a-b)}$$

$\Rightarrow (a+b)^3 = a^3+b^3+3ab(a+b)$

$= a^3+b^3+3a^2b+3ab^2$

या

$a^3+b^3 = (a+b)^3 - 3ab(a+b)$

$\Rightarrow (a-b)^3 = a^3-b^3-3ab(a-b)$

$= a^3-b^3-3a^2b+3ab^2$

या

$a^3-b^3 = (a-b)^3+3ab(a-b)$

$\Rightarrow$ Now $(a+b)^3+(a-b)^3 = a^3+b^3+3a^2b+3ab^2+a^3-b^3-3a^2b+3ab^2$

$= 2a^3+6ab^2$ so →

$\Rightarrow (a+b)^3+(a-b)^3 = 2a(a^2+3b^2)$

$\Rightarrow$ व $(a+b)^3-(a-b)^3 = 2b(b^2+3a^2)$

$\Rightarrow (a^2-b^2) = (a+b)(a-b)$

इसी प्रकार

$a^4-b^4 = (a^2+b^2)(a^2-b^2)$

$= (a^2+b^2)(a+b)(a-b)$

$\Rightarrow a^8-b^8 = (a^4+b^4)(a^2+b^2)(a+b)(a-b)$

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Q:- $\left(24\frac{3}{8}\right)^2 - \left(23\frac{5}{8}\right)^2 = ?$

Sol? $\left(24+\frac{3}{8}\right)^2 - \left(24-\frac{3}{8}\right)^2$

$\Rightarrow 4 \times 24 \times \frac{3}{8} = 36$ ans

$\left[\because (a+b)^2-(a-b)^2 = 4ab\right]$

$\Rightarrow a^2-b^2$

or $\Rightarrow (a+b)(a-b)$

$\Rightarrow \left[24+\frac{3}{8}+23+\frac{5}{8}\right]\left[24\frac{3}{8}-23\frac{5}{8}\right]$

$\Rightarrow 48 \times \frac{6}{8} = 36$ ans

Q:- $24\frac{2}{5} \times 25\frac{3}{5} \Rightarrow ?$

$\Rightarrow \left(25-\frac{3}{5}\right) \times \left(25+\frac{3}{5}\right)$

$\Rightarrow (25)^2 - \left(\frac{3}{5}\right)^2 \Rightarrow 625 - \frac{9}{25}$

$\Rightarrow 624\frac{16}{25}$ ans.

$\Rightarrow (a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow (a+b-c)^2$ or $(c-a-b)^2 = a^2+b^2+c^2+2(ab-bc-ca)$

$\Rightarrow (a-b-c)^2$ or $(b+c-a)^2 = a^2+b^2+c^2+2(bc-ab-ca)$

Note:- $\sqrt{a^2+b^2-2ab} = a-b$ when $a>b$

$= b-a$ when $b>a$.

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$(a^6-b^6) = (a^2)^3-(b^2)^3$

$= (a^2-b^2)(a^4+b^4+a^2b^2)$

$= (a^2-b^2)\left((a^2-b^2)^2+3a^2b^2\right)$ $\because a^2+b^2+ab = (a-b)^2+3ab$

$a^6-b^6 = (a^2-b^2)^3+3a^2b^2(a^2-b^2)$

or

$(a^6-b^6) = (a^3)^2-(b^3)^2$

$= (a^3+b^3)(a^3-b^3)$

$a^6-b^6 = (a-b)(a^2+b^2+ab)(a+b)(a^2+b^2-ab)$

$(a^6+b^6) = (a^2)^3+(b^2)^3$

$= (a^2+b^2)(a^4+b^4-a^2b^2)$ → same as a^2+b^2-ab formula.

$= (a^2+b^2)\left[(a^2+b^2)^2-3a^2b^2\right]$

$a^6+b^6 = (a^2+b^2)^3-3a^2b^2(a^2+b^2)$

By using this we get a Result →

$\sin^6\theta+\cos^6\theta = 1-3\sin^2\theta\cos^2\theta$

$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow (a+b+c)\left[a(a-b)+b(b-c)+c(c-a)\right]$

multiply by $\frac{2}{2}$ and take commons from these →

$= \frac{1}{2}(a+b+c)(2a^2+2b^2+2c^2-2ab-2bc-2ca)$

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Q:- When $a=b=333$ & $c=334$ then value of $a^3+b^3+c^3-3abc$ is ?

$$a^3+b^3+c^3-3abc = \frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

$$= \frac{1}{2}\times 1000 \times \left[0+(-1)^2+(1)^2\right]$$

$$= 1000 \quad \text{ans.}$$

Conclusions → $\dfrac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca} = (a+b+c)$

and $\dfrac{a^2+b^2+c^2-ab-bc-ca}{a^3+b^3+c^3-3abc} = \dfrac{1}{(a+b+c)}$

यदि $a+b+c=0$ हो तो →

$a^3+b^3+c^3-3abc=0 \Rightarrow a^3+b^3+c^3=3abc$

$\dfrac{a^3+b^3+c^3}{3abc}=1$ or $\dfrac{a^3+b^3+c^3}{abc}=3$

$\dfrac{3abc}{a^3+b^3+c^3}=1$ or $\dfrac{abc}{a^3+b^3+c^3}=\dfrac{1}{3}$

यदि $a+b-c=0$ हो तो →

$a^3+b^3-c^3=-3abc$

$\dfrac{a^3+b^3-c^3}{3abc}=-1$ or $\dfrac{a^3+b^3-c^3}{abc}=-3$

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$$\frac{3abc}{a^3+b^3-c^3} = -1 \quad \text{or.} \quad \frac{abc}{a^3+b^3-c^3} = -\frac{1}{3}$$

यदि $a-b-c=0$ हो तो →

$$a^3-b^3-c^3 = 3abc$$

$$\frac{a^3-b^3-c^3}{3abc} = 1 \quad \text{or} \quad \frac{a^3-b^3-c^3}{abc} = 3$$

$$\frac{3abc}{a^3-b^3-c^3} = 1 \quad \text{or.} \quad \frac{abc}{a^3-b^3-c^3} = \frac{1}{3}$$

Q:- $\dfrac{(x-y)^2}{(y-z)(z-x)} + \dfrac{(y-z)^2}{(z-x)(x-y)} + \dfrac{(z-x)^2}{(x-y)(y-z)} = ?$

Soln:→ माना कि $a = x-y$, $b = y-z$ व $c = z-x$ हो तो

यहाँ $\boxed{a+b+c=0}$ so →

$$\Rightarrow \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{a\cdot a^2 + b\cdot b^2 + c\cdot c^2}{abc}$$

$$\Rightarrow \frac{a^3+b^3+c^3}{abc} = 3 \text{ ans.}$$

from condition if $a+b+c=0$ then $\frac{a^3+b^3+c^3}{a\cdot b\cdot c} = 3$

17-2-17

$\left(x+\frac{1}{x}\right) = a$ हो तो $x^2+\frac{1}{x^2} = ?$

$$x^2 + \left(\frac{1}{x}\right)^2 + 2\cancel{x}\cdot\frac{1}{\cancel{x}} = a^2$$

$$x^2+\frac{1}{x^2} = a^2-2$$

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$x - \frac{1}{x} = a$ हो तो $x^2 + \frac{1}{x^2}$ ज्ञात करना ⇒

$$x^2 + \frac{1}{x^2} - 2 \cdot x \cdot \frac{1}{x} = a^2$$

$$x^2 + \frac{1}{x^2} = a^2 + 2$$

Q:- $x - \frac{1}{x} = 0$ तो $x^2 + \frac{1}{x^2} \Rightarrow 0^2 + 2 = \underline{2}$ ans

$\left(x + \frac{1}{x}\right) = a$ हो तो $x^4 + \frac{1}{x^4} = ?$

$$\left(x^2 + \frac{1}{x^2}\right) = a^2 - 2$$

$$\left(x^4 + \frac{1}{x^4}\right) = \underline{(a^2-2)^2 - 2} \Rightarrow x^4 + \frac{1}{x^4} = a^4 - 4a^2 + 2$$

ex.⇒ $x + \frac{1}{x} = 5$ तो $x^4 + \frac{1}{x^4} = ?$

$\Rightarrow (5^2 - 2)^2 - 2 \Rightarrow 529 - 2 = \underline{527}$ ans.

$\left(x - \frac{1}{x}\right) = a$ हो तो $x^4 + \frac{1}{x^4} = ?$

$\because \left(x^2 + \frac{1}{x^2}\right) = a^2 + 2$ तब $x^4 + \frac{1}{x^4} = \underline{(a^2+2)^2 - 2}$

या $\left(x^4 + \frac{1}{x^4}\right) = a^4 + 4a^2 + 2$

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Ex:- $x - \frac{1}{x} = 3$ हो तो $x^4 + \frac{1}{x^4} = ?$

$\Rightarrow (3^2+2)^2 - 2 \Rightarrow 121 - 2 = \underline{119}$ ans.

$x^2 + \frac{1}{x^2} = b$ हो तो $x + \frac{1}{x} = a$ ज्ञात करना →

$x^2 + \frac{1}{x^2} = a^2 - 2 \Rightarrow b = a^2 - 2$

$\Rightarrow a^2 = b + 2 \Rightarrow a = \pm\sqrt{b+2}$

Ex:- $x^2 + \frac{1}{x^2} = 7$ हो तो $x + \frac{1}{x} = ?$

$x + \frac{1}{x} = \pm\sqrt{7+2} \Rightarrow x + \frac{1}{x} = \underline{\pm 3}$ ans.

$x^2 + \frac{1}{x^2} = b$ हो तो $x - \frac{1}{x} = a$ ज्ञात करना →

$x^2 + \frac{1}{x^2} = a^2 + 2 \Rightarrow b = a^2 + 2$

$a^2 = b - 2 \Rightarrow a = \pm\sqrt{b-2}$

Ex:- $x^2 + \frac{1}{x^2} = 6$ हो तो $x - \frac{1}{x} = ?$

$x - \frac{1}{x} = \pm\sqrt{6-2} \Rightarrow x - \frac{1}{x} = \pm 2$ ans.

$x^4 + \frac{1}{x^4} = b$ हो तो $x + \frac{1}{x} = a$ ज्ञात करना →

$x^4 + \frac{1}{x^4} = (a^2-2)^2 - 2 \Rightarrow b = (a^2-2)^2 - 2$

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$(a^2-2)^2 = b+2 \Rightarrow a^2-2 = \sqrt{b+2}$

$\therefore a = \pm\sqrt{\sqrt{b+2}+2}$

Ex:- $x^4+\frac{1}{x^4} = 47$ तो $x+\frac{1}{x} = ?$

$x+\frac{1}{x} = \pm\sqrt{\sqrt{47+2}+2} = \pm\sqrt{7+2} = \pm 3$ ans.

$x^4+\frac{1}{x^4} = b$ हो तो $x-\frac{1}{x} = a$ ज्ञात करना →

$x^4+\frac{1}{x^4} = (a^2+2)^2-2 \Rightarrow b = (a^2+2)^2-2$

$(a^2+2)^2 = b+2. \Rightarrow a = \pm\sqrt{\sqrt{b+2}-2}$

Ex:- $x^4+\frac{1}{x^4} = 119$ हो तो $x-\frac{1}{x} = ?$

$x-\frac{1}{x} = \pm\sqrt{\sqrt{119+2}-2} = \pm\sqrt{11-2}. \Rightarrow a = \pm 3$ ans.

$x+\frac{1}{x} = a$ हो तो $x-\frac{1}{x} = b$ ज्ञात करना →

(M-I) $\left(x+\frac{1}{x}\right)^2-\left(x-\frac{1}{x}\right)^2 = 4$

$\boxed{\sqrt{a^2-4} = b}$

(M-II) $\because x^2+\frac{1}{x^2} = a^2-2 = b^2+2$

$\therefore a^2-2 = b^2+2$

$a^2-4 = b^2$

$\therefore b = \sqrt{a^2-4}$

Ex:- $x+\frac{1}{x} = 5$ तो $x-\frac{1}{x} = \sqrt{a^2-4} = \sqrt{25-4}$

$= \sqrt{21}$ ans

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$x - \frac{1}{x} = a$ हो तो $x + \frac{1}{x} = b$ ज्ञात करना →

$\because \left(x + \frac{1}{x}\right)^2 - \left(x - \frac{1}{x}\right)^2 = 4 \Rightarrow b^2 - a^2 = 4$

$b^2 = a^2 + 4 \Rightarrow \boxed{b = \sqrt{a^2+4}}$

$x + \frac{1}{x} = a$ हो तो $x^3 + \frac{1}{x^3} = ?$

$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3 \cdot x \cdot \frac{1}{x}\left(x + \frac{1}{x}\right)$

$x^3 + \frac{1}{x^3} = a^3 - 3a$

Or

$\left(x + \frac{1}{x}\right)^3 = a^3$

$x^3 + \frac{1}{x^3} + 3 \cdot x \cdot \frac{1}{x}\left(x + \frac{1}{x}\right) = a^3$

$x^3 + \frac{1}{x^3} = a^3 - 3a$

Ex: $x + \frac{1}{x} = 5$ हो तो $x^3 + \frac{1}{x^3} = ?$

$x^3 + \frac{1}{x^3} = 5^3 - 3 \times 5 \Rightarrow 125 - 15 = 110$ ans.

$x - \frac{1}{x} = a$ हो तो $x^3 - \frac{1}{x^3} = ?$

$\left(x - \frac{1}{x}\right)^3 = a^3 \Rightarrow x^3 - \frac{1}{x^3} - 3 \cdot x \cdot \frac{1}{x}\left(x - \frac{1}{x}\right) = a^3$

$x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = a^3$

$x^3 - \frac{1}{x^3} = a^3 + 3a$

Ex: $x - \frac{1}{x} = 5$ हो तो $x^3 - \frac{1}{x^3} = 5^3 + 5 \times 3 \Rightarrow 125 + 15$

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Q:- $\left(x^{100}+\frac{1}{x^{100}}\right)$ (A), $\left(x^{37}+\frac{1}{x^{37}}\right)$ (B) व $\left(x^{100}+\frac{1}{x^{37}}\right)$ (C) का मान क्या होगा

जब (i) $\left(x+\frac{1}{x}\right)=2$ (ii) $\left(x+\frac{1}{x}\right)=-2$ (iii) $\left(x-\frac{1}{x}\right)=0$

Soln:- (i) $x+\frac{1}{x}=2 \Rightarrow x^2+1=2x \Rightarrow x^2-2x+1=0$

$(x-1)^2=0 \Rightarrow x-1=0 \Rightarrow \boxed{x=1}$

x का मान रखने पर →

$\left(x^{100}+\frac{1}{x^{100}}\right)=2$, $\left(x^{37}+\frac{1}{x^{37}}\right)=2$, $\left(x^{100}+\frac{1}{x^{37}}\right)=2$. ans.

(ii) $x+\frac{1}{x}=-2 \Rightarrow x^2+1=-2x \Rightarrow (x+1)^2=0$

$x+1=0 \Rightarrow \boxed{x=-1}$

मान रखने पर

$\left(\frac{(-1)^{100}+1}{(-1)^{100}}\right)=2$, $\left(\frac{(-1)^{37}+1}{(-1)^{37}}\right)=-2$, $\left(\frac{(-1)^{100}+1}{(-1)^{37}}\right)=0$ ans.

(iii) $\left(x-\frac{1}{x}\right)=0 \Rightarrow x^2-1=0 \Rightarrow x^2=1 \Rightarrow \boxed{x=\pm 1}$

$x=+1$ रखने पर →

$A=1+1=2$ $B=1+1=2$ $C=1+1=2$ ans.

$x=-1$ रखने पर

$A=1+1=2$ $B=-1+(-1)=-2$ $C=1-1=0$

ans.

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घातांक व करणी :-

$a^x \rightarrow$ घात
$\hookrightarrow$ आधार

$a^x = b \Rightarrow a = b^{1/x}$ या $a = \sqrt[x]{b}$

$b^{1/x} = \sqrt[x]{b}$ व $b^x = \sqrt[1/x]{b}$ व $b^{y/x} = \sqrt[x/y]{b}$

Ex:→ $x^5 = y \Rightarrow x = y^{1/5}$ या $x = \sqrt[5]{y}$

$a^{x/y} = b \Rightarrow a = b^{y/x}$ या $\sqrt[y/x]{b}$

Ex:- $\sqrt[3/2]{64} = (64)^{2/3} = 16$

$(a^x \pm a^y) = a^x(1 \pm a^{y-x}) = a^y(a^{x-y} \pm 1)$

Ex:- $\dfrac{2^{x+3} + 2^{x-3}}{2^{x+4} + 2^{x-4}} \Rightarrow \dfrac{2^x\left(2^3 + \frac{1}{2^3}\right)}{2^x\left(2^4 + \frac{1}{2^4}\right)} = \dfrac{8 + \frac{1}{8}}{16 + \frac{1}{16}}$

$= \dfrac{65}{8} \times \dfrac{16}{257} = \dfrac{130}{257}$ ans.

$a^x \cdot a^y \cdot a^z \cdot \text{-----} = a^{x+y+z+\text{----}}$

$a^x \cdot b^x \cdot c^x \cdot \text{-----} = (a.b.c.\text{---})^x$

$a^x \div a^y = \dfrac{a^x}{a^y} = a^{x-y} = \dfrac{1}{a^{y-x}}$

$a^x \div b^x = \left(\dfrac{a}{b}\right)^x$ | $(a^x)^y = a^{xy} = a^{yx}$

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NOTE:- a^{x^y} में यदि bracket नहीं लगा हुआ है तो पहले x^y solve होगा।

Ex:- $2^{2^3} = 2^8 = 256$

but $(2^3)^2 = 2^6 = 64$ होगा

$2^{3^2} = 2^9 = 512$

$\sqrt[x]{a} \times \sqrt[y]{a} = a^{1/x} \cdot a^{1/y} = \sqrt[\frac{xy}{x+y}]{a}$

$\sqrt[x]{a} \div \sqrt[y]{a} = a^{1/x} \times a^{-1/y} = a^{\frac{1}{x}-\frac{1}{y}} = a^{\frac{y-x}{xy}} = \sqrt[\frac{xy}{y-x}]{a}$

$\sqrt[x]{a} \cdot \sqrt[x]{b} \cdot \sqrt[x]{c} \cdot \text{---} = \sqrt[x]{a \times b \times c \text{---}}$

$\sqrt[x]{a} \div \sqrt[x]{b} = \frac{\sqrt[x]{a}}{\sqrt[x]{b}} = \sqrt[x]{\frac{a}{b}}$

$\sqrt[y]{a^x} = a^{x/y} = \sqrt[y/x]{a}$

$a^1 = a$, $1^x = 1$, $a^0 = 1$

$(\sqrt[b]{a})^x = a^{x/y} = \sqrt[y/x]{a}$

$\sqrt[x]{a^x} = (\sqrt[x]{a})^x = a^{x/x} = a^1 = a$

जब $n \rightarrow$ प्राकृत संख्या, $2n \rightarrow$ सम संख्या व $2n \pm 1 \rightarrow$ विषम संख्या हो

तो $(a^{2n} - b^{2n})$ के factors $\rightarrow (a-b)$, $(a+b)$

$(a^{2n+1} - b^{2n+1})$ हमेशा $\rightarrow (a-b)$ से divisible होगा।

$(a^{2n+1} + b^{2n+1})$ हमेशा $\rightarrow (a+b)$ से divisible होगा।

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Ex:- 7^3+6^3 → always divisible by $(7+6)=13$.

Q:- $67^{67}+67$ को 68 से भाग दे तो शेषफल क्या होगा ?

Soln:- $67^{67}+67 \Rightarrow (67^{67}+1^{67})+66$ अतः शेषफल = 66 ans.

↓ divisible by 67+1=68

Ex:- $123^{123}+123$ में 124 का भाग देने पर शेषफल = 122 ans.

a^{2n} ⇒ $(a-1)$ से भाग दे तो शेष = 1
$(a+1)$ से भाग दे तो शेष = 1

Ex:- 17^{200} में 16 का भाग दे तो शेष = 1
" 18 " " शेष = 1

$a^{(2n+1)}$ में $(a-1)$ से भाग दे तो शेषफल = 1
$(a+1)$ से " " = a

Ex: 17^{99} में 16 से भाग दे तो शेष = 1
18 से भाग दे तो शेष = 17

$(a^{2n+1}-1)$ divided by $(a-1)$

$(a^{2n+1}-a)$ divided by $(a+1)$

Ex:- $(17^{201}-1)$ ⟶ 16 से divide होगी।

$a(a^{2n}-1)$ ⟶ $a(a-1)(a+1)$ से divide होगी।

Ex:- $17(17^{200}-1)$ ⟶ 16, 17, 18 से divide होगी।

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$(10^3 - 10)$ ⟶ 9, 10, 11 से divide होगा ।

$(12^5 - 12)$ ⟶ 11, 12, 13 से divide होगा ।

Rational & Irrational Numbers / परिमेय व अपरिमेय संख्याये

$\frac{p}{q}$ एक परिमेय संख्या होगी जहाँ $\boxed{q \neq 0}$

$\sqrt{2} = 1.4142$, $\sqrt{3} = 1.732$

π → अपरिमेय संख्या है जबकि $\frac{22}{7}$ परिमेय ।

$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a}$, $\left(\frac{1}{\sqrt{a}+\sqrt{b}}\right) = \frac{\sqrt{a}-\sqrt{b}}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})} = \frac{\sqrt{a}-\sqrt{b}}{a-b}$

$\left(\frac{1}{\sqrt{a}-\sqrt{b}}\right) = \frac{(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})} = \frac{(\sqrt{a}+\sqrt{b})}{a-b}$

$\left(\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right) = \frac{(\sqrt{a}+\sqrt{b})(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})} = \frac{(\sqrt{a}+\sqrt{b})^2}{a-b} = \frac{a+b+2\sqrt{ab}}{a-b}$

$\left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}\right) = \frac{(\sqrt{a}-\sqrt{b})(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})} = \frac{(\sqrt{a}-\sqrt{b})^2}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})} = \frac{a+b-2\sqrt{ab}}{a-b}$

$\left(\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right) + \left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}\right) = \frac{(\sqrt{a}+\sqrt{b})^2 + (\sqrt{a}-\sqrt{b})^2}{a-b} = \frac{2(a+b)}{a-b}$

$\left(\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right) - \left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}\right) = \frac{(\sqrt{a}+\sqrt{b})^2 - (\sqrt{a}-\sqrt{b})^2}{a-b} = \frac{4\sqrt{ab}}{a-b}$

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$\because (a+b)(a^2+b^2-ab) = a^3+b^3$ then

\# $\left(\sqrt[3]{a}+\sqrt[3]{b}\right)\left[\left(\sqrt[3]{a}\right)^2+\left(\sqrt[3]{b}\right)^2-\sqrt[3]{ab}\right] = a+b$

or

$\left(\sqrt[3]{a}+\sqrt[3]{b}\right)\left(\sqrt[3]{a^2}+\sqrt[3]{b^2}-\sqrt[3]{ab}\right) = (a+b)$

\# $\frac{1}{\left(\sqrt[3]{a}+\sqrt[3]{b}-\sqrt[3]{ab}\right)} = \left(\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}-\sqrt[3]{ab}}\right)\times\left(\frac{\sqrt[3]{a}+\sqrt[3]{b}}{\sqrt[3]{a}+\sqrt[3]{b}}\right) = \frac{\sqrt[3]{a}+\sqrt[3]{b}}{a+b}$

$\because (a^2+b^2+ab)(a-b) = a^3-b^3$ then

$\left(\sqrt[3]{a^2}+\sqrt[3]{b^2}+\sqrt[3]{ab}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}\right) = a-b$

इसी प्रकार →

$$\frac{1}{\left(\sqrt[3]{a^2}+\sqrt[3]{b^2}+\sqrt[3]{ab}\right)} = \frac{\sqrt[3]{a}-\sqrt[3]{b}}{a-b}$$

\# $(\sqrt{a}+\sqrt{b})^2 = a+b+2\sqrt{ab}$ | $(\sqrt{a}-\sqrt{b})^2 = a+b-2\sqrt{ab}$

\# $\sqrt{a+b+2\sqrt{ab}} = \sqrt{a}+\sqrt{b}$

\# $\sqrt{a+b-2\sqrt{ab}} = (\sqrt{a}-\sqrt{b})$ when $a>b$

$\Rightarrow (\sqrt{b}-\sqrt{a})$ $b>a$

Ex:→ $\sqrt{7+2\sqrt{12}} = \sqrt{3}+\sqrt{4}$, $\sqrt{7-2\sqrt{12}} = \sqrt{4}-\sqrt{3}$

$\sqrt{\sqrt{81}+\sqrt{80}} = \sqrt{9+\sqrt{81}} = \sqrt{4}+\sqrt{5} = 2+\sqrt{5}$

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योगान्तरानुपात :- (योग : अन्तर)

$(a, b) \rightarrow (a+b) : (a-b)$ or $\frac{a+b}{a-b}$

① यदि $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ है तो $\frac{a}{b} = \frac{c}{a}$ होगा।

② $\frac{a+b}{a-b} = \frac{c}{d}$ है तो $\frac{a}{b} = \frac{c+d}{c-d}$ होगा।

③ $\frac{a}{b} = \frac{c+d}{c-d}$ है तो $\frac{a+b}{a-b} = \frac{c}{d}$ होगा।

Equations / समीकरण (रेखीय समीकरण)

$Ax + By + C = 0$

if $a_1x + b_1x + c_1 = 0$ व & $a_2x + b_2x + c_2 = 0$ दो line की समी. हो तो -

यदि $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ होतो Lines अतिव्यापी / overlaping / आच्छादी होंगी

प्रतिच्छेद बिंदु ⇒ अनंत

हल → असीम / अनंत हल होंगे।

यदि $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ हो तो रेखायें - समान्तर होगी।

या अप्रतिच्छेदी रेखायें होंगी।

⇒ कोई प्रतिच्छेद बिंदु नहीं व कोई solution नहीं होगा।

Ex: $-3x + 4y + 5 = 0$
$9x + 12y + 8 = 0$ रेखायें समान्तर होगी

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यदि $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ हो तो रेखायें प्रतिच्छेदी होगी।

दोनों एक निश्चित बिन्दु (x, y) पर प्रतिच्छेद करती है। व हल निश्चित / अद्वितीय / अनन्य होते हैं

जब दो सरल रेखायें परस्पर लम्बवत प्रतिच्छेदी हो तो -

$$\boxed{a_1a_2 = -b_1b_2} \quad \text{or} \quad \boxed{a_1a_2 + b_1b_2 = 0}$$

Ex. $3x + 4y + 15 = 0$ यहाँ $a_1a_2 = 3 \times 12 = 36$

$12x - 9y + 7 = 0$ $b_1b_2 = +4 \times -9 = -36$

अतः $a_1a_2 + b_1b_2 = 0$ है so एक दूसरे को प्रतिच्छेद करेगी।

द्विघात समीकरण :- $ax^2 + bx + c = 0$

Ex ① $x^2 - 5x + 6 = 0$

$x^2 - 3x - 2x + 6 = 0$

$x(x-3) - 2(x-3) = 0$

$(x-2)(x-3) = 0$

$x = 2, 3$

② $x^2 + 5x + 6 = 0$

$x^2 + 2x + 3x + 6 = 0$

$x(x+2) + 3(x+2) = 0$

$(x+2)(x+3) = 0$

$x = -2, -3$

③ $x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$x(x-3) + 2(x-3) = 0$

$(x+2)(x-3) = 0$

$x = -2, 3$

④ $x^2 + x - 6 = 0$

$x^2 + 3x - 2x - 6 = 0$

$x(x+3) - 2(x+3) = 0$

$(x-2)(x+3) = 0$

$x = 2, -3$

यहाँ a, b समी. $ax^2 + bx + c$ के solution / हल / शून्यांक कहलाते हैं।

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$D = b^2 - 4ac$ → विविक्तकर / Dis criminato

$\alpha/\beta = \frac{-b \pm \sqrt{D}}{2a}$ या $\alpha/\beta = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

"श्रीधराचार्य सूत्र"

मूलों की प्रकृति :-

(i) जब $D = 0$ or $b^2 - 4ac = 0$ हो या $b^2 = 4ac$ हो
तो मूल → Real (वास्तविक)
Rational (परिमेय)
Equal (समान) होंगे।
व
मूलों की value → $\alpha = \beta = \frac{-b}{2a}$ होगी।

Ex:- $x^2 - 6x + 9 = 0$
$D = b^2 - 4ac = (-6)^2 - 4 \times 1 \times 9 = 36 - 36 = 0$
$\boxed{D=0}$ अतः मूल समान $\alpha = \beta = \frac{-b}{2a} = \frac{-(-6)}{2\times1} = 3$ होंगे। ans.

(ii) जब $D > 0$ व एक पूर्ण वर्ग संख्या है तब -
मूल → Real (वास्तविक)
Rational (परिमेय)
Inequal (असमान) होंगे।

(iii) जब $D > 0$ व पूर्ण वर्ग संख्या नहीं है तब
मूल → Real (वास्तविक)
Irrational (अपरिमेय)
Inequal (असमान) होंगे।

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(iv) जब $D < 0$ or D, -ve हो तब →

मूल ⟶ Imaginary (काल्पनिक)

Inequal (असमान) होंगे।

मूलों का योग $\alpha+\beta = \left(\frac{-b}{a}\right)$, मूलों का गुणनफल $\alpha\beta = \left(\frac{c}{a}\right)$

Ex: यदि समी० $x^2-5x+6=0$ हो तो

$$\alpha+\beta = \left(\frac{-b}{a}\right) = \frac{-(-5)}{1} = 5$$

व $\alpha\beta = \left(\frac{c}{a}\right) = \frac{6}{1} = 6$ ans

यदि α व β किसी समी० के मूल हो तो वह समी० →

$$x^2-(\alpha+\beta)x+\alpha\beta = 0$$

Q:- x^2-x+1 का न्यूनतम मान ज्ञात करो।

Soln:- $x^2-x+1 \Rightarrow x^2-x+\frac{1}{4}+\frac{3}{4}$

$\Rightarrow \left(x-\frac{1}{2}\right)^2+\frac{3}{4}$ ∵ पूर्ण वर्ग का न्यूनतम मान = 0

$\Rightarrow 0+\frac{3}{4} = \frac{3}{4}$ ans.

Q:- x^2+4x+1 का न्यूनतम मान = ?

$x^2+4x+1 \Rightarrow (x^2+4x+4)-3 \Rightarrow (x+2)^2-3$

$\Rightarrow 0-3 = -3$ ans.

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ax^2+bx+c का न्यूनतम मान $= \frac{4ac-b^2}{4a}$

Ex: $x^2-x+1 \Rightarrow \frac{4\times1\times1-1}{4\times1} = \frac{4-1}{4} = \frac{3}{4}$

व $x^2+4x+1 \Rightarrow \frac{4\times1\times1-16}{4\times1} = \frac{-12}{4} = -3$ ans.

न्यूनतम मान पर x की value $x = \frac{-b}{2a}$ होगी।

ax^2+bx+c का अधिकतम मान $= \infty$ (अनंत)

जब x^2 का गुणांक (-ve) हो तब अधिकतम मान $= \frac{4ac-b^2}{4a}$

Ex: $\frac{1}{x^2-x+1}$ का अधिकतम मान $= \frac{1}{x^2-x+1 \text{ का न्यूनतम}} = \frac{1}{3/4} = \frac{4}{3}$ ans

Q:- $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots\cdots}}}} = ?$ $= x$

Soln:- $2+x = x^2 \Rightarrow x^2-x-2=0$

$x^2-2x+x-2=0$

$x(x-2)+1(x-2)=0$

$(x+1)(x-2)=0$ So $x=-1, 2$

Root का मान -ve नहीं होता अत $\boxed{x=2}$ ans

Q:- $\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2-\cdots\cdots}}}}\ \infty = ?$ Let $x=?$

Soln:- $2-x = x^2 \Rightarrow x^2+x-2=0 \Rightarrow x^2+2x-x-2=0$

$x(x+2)-1(x+2)=0 \Rightarrow (x+2)(x-1)=0 \Rightarrow x=+1, -2$

+ve answer use करने पर $\boxed{x=1}$ ans.

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NOTE :- Roots में संख्यायें $n \times (n+1)$ के रूप में चलती है तो (+ve) sign का Repetition होने पर ans → $(n+1)$
व (−ve) sign " " ans → n होगा ।

for Ex :- $\sqrt{20+\sqrt{20+\sqrt{20+\cdots\infty}}} = ?$

यहाँ $20 = 4 \times 5$ व sign +ve है अतः ans = 5 होगा

यदि $\sqrt{x+\sqrt{x+\sqrt{x+\cdots\infty}}} = a$ हो तो

then- $\boxed{x = a \times (a-1)}$ होगा ।

यदि $\sqrt{x-\sqrt{x-\sqrt{x-\cdots\infty}}} = a$ हो तो

$\boxed{x = a \times (a+1)}$ होगा ।

Ex :- $\sqrt{x+\sqrt{x+\sqrt{x+\cdots\infty}}} = 12$ then $x = ?$

$x = 12 \times (12-1) = 12 \times 11 = 132$ ans.

Ex :- $\sqrt{4+\sqrt{4+\sqrt{4+\cdots\infty}}} = ?(x)$

(A) $1 > x > 0$ (B) $2 > x > 1$ (C) $3 > x > 2$ (D) $x = 2$

$\sqrt{x \div \sqrt{x \div \sqrt{x \div \sqrt{x \div \cdots\infty}}}}$ (a)

$x \div a = a^2$

$x = a^3$

$\boxed{a = \sqrt[3]{x}}$ ans

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$\sqrt{x\sqrt{x\sqrt{x\sqrt{x}}}}\cdots\infty = x$

$\sqrt{x\sqrt{x\sqrt{x}}}\cdots n = x^{\left(\frac{2^n-1}{2^n}\right)}$

Ex:- $\sqrt{x}$ $n=1 \longrightarrow x^{1/2}$ $\quad n=8 \longrightarrow x^{\left(\frac{255}{256}\right)}$

$\sqrt{x\sqrt{x}}$ $n=2 \longrightarrow x^{3/4}$ $\quad n=9 \longrightarrow x^{\left(\frac{511}{512}\right)}$

$\sqrt{x\sqrt{x\sqrt{x}}}$ $n=3 \longrightarrow x^{7/8}$ $\quad n=10 \longrightarrow x^{\left(\frac{1023}{1024}\right)}$

$n=4 \longrightarrow x^{(15/16)}$ $\quad n=11 \longrightarrow x^{\left(\frac{2047}{2048}\right)}$

$n=5 \longrightarrow x^{(31/32)}$ $\quad n=12 \longrightarrow x^{\left(\frac{4095}{4096}\right)}$

$n=6 \longrightarrow x^{\left(\frac{63}{64}\right)}$ $\quad n=13 \longrightarrow x^{\left(\frac{8191}{8192}\right)}$

$n=7 \longrightarrow x^{\left(\frac{127}{128}\right)}$

Q:→ $\sqrt{x \div \sqrt{x \div \sqrt{x \div}}}\cdots = ?$ Let $a = ?$

Sol^{n}:→ $x \div a = a^2$

$x = a3 \Rightarrow \boxed{a = \sqrt[3]{x}}$ ans.

NOTE :- यदि $a^2+b^2+c^2 = ab+bc+ca$ हो तो यह तभी संभव है जब $\boxed{a=b=c}$ हो।

OR $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} = \frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}$

तभी संभव है जब $\boxed{a=b=c}$

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